Conditional Multivariate Risk Measures

Beatriz Vaz de Melo Mendes [1]

IM/COPPEAD, Federal University at Rio de Janeiro, Brazil.

Abstract

Models for extreme joint tails date back to Tiago de Oliveira (1962), Pickands (1981), Tawn (1988), and are based on limiting arguments founded on multivariate regular variation. All these models, including extreme value copulas, are designed for asymptotically dependent variables, and assume that all components become large at the same rate. Heffernan and Tawn (2004) proposed a conditional multivariate extreme value model which applies to regions where not all variables are extreme and identifies the type of extremal dependence, including negative dependence. In this paper we exploit this work and provide an application in finance. The new methodology allows for estimating new measures of financial risk, namely the Conditional Value-at-Risk and the Conditional Expected Shortfall given that at least one of the data components is extreme, and provides further information for portfolio selection and risk management. We illustrate using Latin American and Asian markets indexes, with interesting findings which are consistent but goes beyond the current understanding of the interdependencies in these emerging markets.

Keywords: Conditional Multivariate Extreme Value Models, Asymptotic Independence, Extremal Dependence, Financial Risk.

1 Introduction

Models for extreme tails date back to Tiago de Oliveira (1962). The existing multivariate extreme value models (Pickands (1981), Tawn (1988, 1990), Coles and Tawn (1991, 1994), and Joe, Smith, and Weissman (1994), among others) assume that in some extreme joint tail region all components are either independent or asymptotically dependent. Within the copula environment, these two cases are captured by extreme value copulas possessing either zero or positive tail dependence coefficient. All these models are supported by limiting arguments based on the concept of multivariate regular variation, and assume that all margins become extreme at the same rate.

Under the existing multivariate extreme value models the probability of joint extreme events may be under estimated. Note that Ledford and Tawn (1996) have identified three types of extremal dependence (negative, independence, positive) in the case of asymptotically independent variables. Moreover, a multivariate extreme event may be extreme in just one of the components. Heffernan and Tawn (2004) proposed a modeling structure

[1]Email: beatriz@im.ufrj.br

which addresses these two issues. In summary, the model applies to regions where not all variables are extreme, and covers the three types of extremal dependence.

Applications of multivariate extreme value models include environment impact assessment (Coles and Tawn (1994), Smith (1989), among others), financial risks management (Stärica (2000), McNeil (1999), among others). In finance, the implications of multivariate extreme events are felt in portfolio behavior, asset pricing, and may lead to a market crash, bankruptcy, and defaults. In this article we exploit Heffernan and Tawn (2004) conditional model and provide an application in finance. The new methodology allows for estimating new measures of financial risk, conditional on at least one of the data components is extreme, namely the Conditional Value-at-Risk and the Conditional Expected Shortfall. One appealing characteristic of the new measures of risk is that they may incorporate intangibles in the notion of contagion.

Our empirical illustration uses emerging markets data. For the five major market indexes in Latin America and five ones in Asia, we estimate the conditional models, analyze the type of dependence among the components, compare the influence of the conditioning market, compute measures of dependence, discuss the main differences between the unconditional and conditional distributions, compute the new risk measures, and finally use all acquired knowledge to suggest other financial applications such as portfolio selection, showing the impact of the estimated conditional dependence structure on portfolio risk an return.

The news brought by the estimated model adds knowledge to the current understanding of interdependence among emerging markets. For example, we observe that the worst scenarios for Brazil occur when either Chile or U.S. is extreme, and the worst scenarios for the U.S. occur when either Brazil or Mexico is extreme. Somehow expected is the finding that the only non-exchangeable pairs of joint losses in Latin America are those involving the U.S. market. From the empirical investigations we could say that in order to diversify portfolios one should rather include a variable which does not conditionally drives dependence during bear markets. However, one should also take into account the marginal distribution of this variable, since risk (or VaR) may increase with the addition of this new variable. In summary, both marginal and conditional multivariate knowledge of the variables are important in risk management.

The remaining sections are organized as follows: Section 2 reviews the Heffernan and Tawn (2004) conditional extreme value model. Section 3 contains the analysis of the ten emerging markets indexes, including marginal and dependence models estimation, computation of risk measures, and illustrations of applications in finance. In Section 4 we discuss the results.

2 The conditional extreme value model

In this section we briefly explain the Heffernan and Tawn (2004) conditional model.

As already mentioned, existing extreme value methods for VaR estimation are based on the assumption that all variables become extreme at the same rate. The model used in this paper applies to situations where at least one variable is extreme. This situation is illustrated in Figure 1 based on pairwise monthly minima from main indexes from Argentina, Chile, and U.S. Plot (a) shows the observations from the pair (U.S.-Chile) along with the extreme set C, shown in the figure by the dotted line. Plot (b) shows the observations from the pair (Argentina-Chile), which are clearly asymptotically independent. For example, it would be interesting to assess what happens to the pair (Argentina-Chile) when U.S. is extreme or falls in the region marked with a ? in the set C.

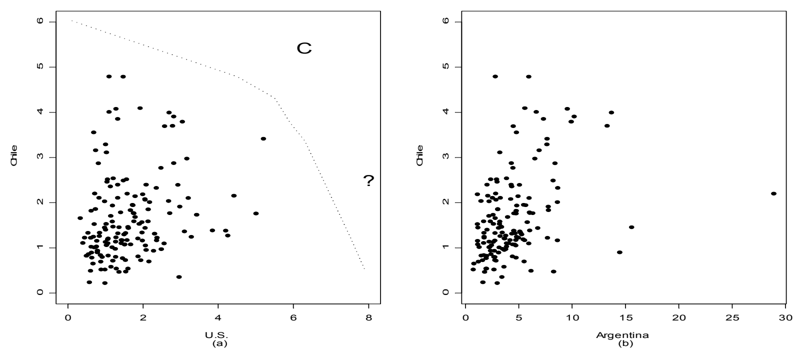

Figure 1: *Monthly minima of U.S. and Chile in* (a) *and from Argentina and Chile in* (b). *The dotted line in* (a) *indicates an extreme set* C.

The multivariate extreme value model of Heffernan and Tawn (2004) may be summarized as follows. Consider a continuous d-dimensional random variable $\mathbf{X} = (X_1, \cdots, X_d)$ with unknown distribution $F(\mathbf{x})$. We wish to estimate functionals of the distribution of $\mathbf{X}$ when $\mathbf{X}$ is extreme in at least one component, based on a sample of n independent and identically distributed observations from F. This means to estimate the probabilities $\Pr(\mathbf{X} \in C)$ where C is an extreme set such that for all $\mathbf{x} \in C$ at least one component of $\mathbf{x}$ is extreme. The set C is partioned into d subsets C_i such that $C = \cup_{i=1}^{d} C_i$. The subset C_i is the part of C for which X_i is the largest component of $\mathbf{X}$.

Let F_{X_i} denote the marginal distribution of X_i, $i = 1, \cdots, d$. Then

$$C_i = C \cap \{\mathbf{x} \in \Re^d : F_{X_i}(x_i) > F_{X_j}(x_j); j = 1, \cdots, d; j \neq i\}, \qquad \text{for} \quad i = 1, \cdots, d.$$

Thus C is an extreme set if it is the disjoint union of subsets C_i which are either empty or have points $\mathbf{x}$ such that their x_i-values fall in the upper tail of F_{X_i}. That is, if $\nu_{X_i} = \inf_{\mathbf{x}\in C_i}(x_i)$, then $F_{X_i}(\nu_{X_i})$ is close to 1. Thus $\mathbf{X} \in C_i$ if and only if $\mathbf{X} \in C_i$ and $X_i > \nu_{X_i}$, so we can write

$$\Pr(\mathbf{X} \in C) = \sum_{i=1}^{d} \Pr(\mathbf{X} \in C_i) = \sum_{i=1}^{d} \Pr(\mathbf{X} \in C_i \mid X_i > \nu_{X_i}) \Pr(X_i > \nu_{X_i}). \tag{1}$$

Computing the probabilities (1) requires a marginal extreme value model for $\Pr(X_i > \nu_{X_i})$ and an extreme value model for the dependence structure $\Pr(\mathbf{X} \in C_i \mid X_i > \nu_{X_i})$. According to results in Pickands (1975), the generalized Pareto distribution (GPD) is the appropriate model for excesses beyond a high threshold. The marginal model for the tail of X_i, $i = 1, \cdots, d$, is then

$$\Pr(X_i > x + u_{X_i} \mid X_i > u_{X_i}) = (1 + \xi_i x/\beta_i)_+^{-1/\xi_i} \quad \textit{where} \quad x > 0, \tag{2}$$

and where u_{X_i} is a high threshold for the variable X_i, $\beta_i > 0$ and ξ_i are the scale and shape parameters of the GPD, and $s_+ = \max(s, 0)$ for any $s \in \Re$. To model the marginal distribution F_{X_i} we assume the semiparametric model $\widehat{F}_{X_i}$ given by

$$\widehat{F}_{X_i} = \begin{cases} 1 - \{1 - \tilde{F}_{X_i}(u_{X_i})\}\{1 + \xi_i(x - u_{X_i})/\beta_i\}_+^{-1/\xi_i} & \text{for} \quad x > u_{X_i}, \\ \tilde{F}_{X_i}(x) & \text{for} \quad x \leq u_{X_i}, \end{cases} \tag{3}$$

where $\tilde{F}_{X_i}$ is the empirical distribution of the X_i-values. Model (3) is used for estimating the term $\Pr(X_i > \nu_{X-i})$ in the decomposition (1).

Before deriving the conditional dependence model, the univariate marginals are transformed to standard Gumbel distributions, by applying the probability integral transformation to (3):

$$Y_i = -\log[-\log\{\widehat{F}_{X_i}(X_i)\}] \quad \text{for} \quad i = 1, \cdots, d, \tag{4}$$

where $\widehat{F}_{X_i}$ depend on the marginal parameters (β_i, ξ_i). As discussed by Heffernan and Tawn (2004) this transformation is supported by the fact that a Gumbel random variable has an exponential upper tail. From now on the notation $\mathbf{Y}$ refers to the Gumbel transformation of the original $\mathbf{X}$ variables.

Let $\mathbf{Y}_{-i}$ denote the vector $\mathbf{Y}$ excluding the component Y_i, and $\mathbf{y}$ a vector of y-values. We condition on each variable in turn and look at the limiting conditional distribution of

$$\mathbf{Y}_{-i} \mid Y_i > y_i \quad \text{as} \quad y_i \to \infty.$$

In the limit, the random variable $\mathbf{Y}_{-i}$ may be either (asymptotically) dependent or independent of the variable Y_i. All existing methods for multivariate extremes, including extreme value copulas, apply when the associated $\mathbf{Y}$ is asymptotically dependent, or when the sets C of interest are such that all $\mathbf{x} \in C$ are large in all components. The conditional Heffernan and Tawn (2004) model may be applied when the variables are asymptotically dependent or independent and may be interpreted as an extension of the GPD to the multivariate case.

In summary, the multivariate models consist of a parametric regression, carried on to estimate the location and the scale parameters of the marginals of the joint conditional distribution, and a nonparametric method, used to estimate the multivariate limit structure. Likewise the existing multivariate extreme value models, the idea is that, for large conditioning values, the conditional distribution of the properly standardized variables would possess some specified structure. However, a complete asymptotic characterization of the probabilist structure is not obtained, but it is non-parametrically inferred from the standardized variables.

As usual, it is required that the limiting conditional distribution $\Pr(\mathbf{Y}_{-i} \leq \mathbf{y}_{-i} \mid Y_i = y_i)$ as $y_i \to \infty$ be non-degenerate in all margins. This is accomplished by assuming a vector of normalizing functions holds for each i. The normalizing functions $\mathbf{a}_{|i}(y_i)$ and $\mathbf{b}_{|i}(y_i)$, both in $\Re \to \Re^{(d-1)}$ are chosen such that, for all fixed $\mathbf{z}_{-i}$ and for any sequence of y_i-values such that $y_i \to \infty$, it holds

$$\lim_{y_i \to \infty} \Pr\{\mathbf{Y}_{-i} \leq \mathbf{a}_{|i}(y_i) + \mathbf{b}_{|i}(y_i)\mathbf{z}_{-i} \mid Y_i = y_i\} = G_{|i}(\mathbf{z}_{-i}), \tag{5}$$

where all margins of the limit distribution $G_{|i}$ are non-degenerate.

It can be shown that under assumption (5), conditionally on $Y_i > u_i$, as $u_i \to \infty$ the variables $Y_i - u_i$ and $\mathbf{Z}_{-i}$ are independent in the limit with limiting marginal distributions being, respectively, the exponential and $G_{|i}(\mathbf{z}_{-i})$.

Following standard arguments (Leadbetter et al., 1983) the authors identify the normalizing functions $\mathbf{a}_{|i}(y_i)$ and $\mathbf{b}_{|i}(y_i)$ (up to some vector constants), and show that the class of limit distributions is unique up to type (see Theorem 1 and Corollary 1 in Heffernan and Tawn (2004). Through theoretical examples the authors show that based on a simple structure for the normalizing constants, a wide range of limiting distributions $G_{|i}$ can be found which could not be contained in any simple distributional family. This finding about $G_{|i}$ is in contrast with the limiting representation for multivariate extreme value distributions, but is was expected, due the lack of structure imposed on $G_{|i}$ by the limiting operation. They proposed the following general parametric family for the normalizing functions:

$$\begin{aligned} &\mathbf{a}_{|i}(y) = \mathbf{a}_{|i} y + \mathbf{I}_{\{\mathbf{a}_{|i}=\mathbf{0}, \mathbf{b}_{|i}<\mathbf{0}\}}\{\mathbf{c}_{|i} - \mathbf{d}_{|i} \log(y)\} \quad , \\ &\mathbf{b}_{|i}(y) = y^{\mathbf{b}_{|i}} \quad , \end{aligned} \tag{6}$$

where, $\mathbf{a}_{|i}$, $\mathbf{b}_{|i}$, $\mathbf{c}_{|i}$, and $\mathbf{d}_{|i}$ are vector constants and $\mathbf{I}$ is an indicator function. Their components must satisfy $0 \leq a_{j|i} \leq 1$, $-\infty < b_{j|i} < 1$, $-\infty < c_{j|i} < \infty$, $0 \leq d_{j|i} \leq 1$, for all $j \neq i$.

It is assumed that there is a high threshold u_{Y_i} for which the model (5) holds for all $y_i > u_{Y_i}$. $\mathbf{Z}_{-i} = \frac{\mathbf{Y}_{-i} - \mathbf{a}_{|i}(y_i)}{\mathbf{b}_{|i}(y_i)}$ is the standardized variable with $(d-1)$-dimensional distribution function $G_{|i}$, and $\mathbf{Z}_{-i}$ is independent of Y_i for $Y_i > u_{Y_i}$. The extremal dependence behavior is characterized by the location and scale functions $\mathbf{a}_{|i}(y_i)$ and $\mathbf{b}_{|i}(y_i)$ and the distribution $G_{|i}$. For the sake of simplicity, for estimation purposes, we start with the assumption that $\mathbf{c}_{|i}(y_i) = \mathbf{d}_{|i}(y_i) = 0$, and then estimates these parameters whenever $\hat{\mathbf{a}}_{|i}(y_i) = 0$ and $\hat{\mathbf{b}}_{|i}(y_i) < 0$. To (non-parametrically) estimate $G_{|i}$ we use the empirical distribution of the replicates of $\hat{\mathbf{Z}}_{-i}$. In summary, for $d = 1, \cdots, d$ our dependence model is is a multivariate semi-parametric regression model of the form

$$\mathbf{Y}_{-i} = \mathbf{a}_{|i}(y_i) + \mathbf{b}_{|i}(y_i)\mathbf{Z}_{-i} \qquad \text{for} \;\; Y_i = y_i.u_{Y_i} \;, \tag{7}$$

where $\mathbf{a}_{|i}(y_i)$ and $\mathbf{b}_{|i}(y_i)$ are given by the parametric model (6), and the distribution of the standardized variable is modeled non-parametrically. It is not required that the dependence threshold u_{Y_i} and u_{X_i} to agree in the sense that $u_{Y_i} = t_i(u_{X_i})$, where t_i is the Gumbel transformation (equation (3.8) of Heffernan and Tawn (2004)).

Four classes of dependence structures are implied by (7). If $a_{j|i} = 1$ and $b_{j|i} = 0$, the quantiles of the variables (Y_i, Y_j) grow at the same rate, and they are asymptotically dependent. All other possibilities for $a_{j|i}$ and $b_{j|i}$ correspond to the case of asymptotic independence. Within the class of asymptotically independent variables Ledford and Tawn (1996) identify three classes: positive extremal dependence (at least one of $0 < a_{j|i} < 1$ or $b_{j|i} > 0$ holds), near extremal independence ($a_{j|i} = d_{j|i} = 0$ and $b_{j|i} \leq 0$ holds), and negative extremal dependence ($a_{j|i} = 0$, $d_{j|i} > 0$, and $b_{j|i} < 0$ holds). For these cases, as $y_i \to \infty$, the quantiles of the distribution of $Y_j \mid Y_i = y_i$ tend to ∞, to a finite limit, or to $-\infty$, respectively. The $c_{j|i}$ and $d_{j|i}$ are non-zero only in the case that there is *no* positive association.

There is *weak* pairwise extremal exchangeability $a_{j|i} = a_{i|j}$, $b_{j|i} = b_{i|j}$, $c_{j|i} = c_{i|j}$, and $d_{j|i} = d_{i|j}$. For each i, Heffernan and Tawn (2004) approach is to estimate the d different conditional distributions separately, and not to impose additional structure on (7). They recommend assessing the effect of using different conditionals to estimate probabilities of events in which more that one variable is extreme, and to average estimates over the different conditionals to reduce any problem of inconsistency.

Inference is carried on in two steps: first the marginal parameters are estimated, and then the dependence parameters are estimated assuming that the marginal parameters are known. Assuming independence between the d components we maximize the log-likelihood function

$$\sum_{i=1}^{d} \sum_{k=1}^{n_{u_{X_i}}} \log\{\hat{f}_{X_i}(x_{j|i,k})\} \quad j = 1, \cdots, d, \tag{8}$$

where $\hat{f}_{X_i}$ is the density of distribution (3), $n_{u_{X_i}}$ is the number of observations with ith component exceeding the threshold u_{X_i}, and the jth component of the kth such observation is denoted by $x_{j|i,k}$, $j = 1, \cdots, d$, $k = 1, \cdots, n_{u_{X_i}}$. In the case there is no links between the parameters of the components, maximization of (8) may be carried on by obtaining the maximum likelihood estimates of the GPD fitted to the excesses, in each margin.

For each i, to estimate $(\mathbf{a}_{|i}, \mathbf{b}_{|i}, \mathbf{c}_{|i}, \mathbf{d}_{|i})$, we assume that the $\mathbf{Z}_{-i}$ have marginal means and standard deviations, respectively, $\boldsymbol{\mu}_{|i}$ and $\boldsymbol{\sigma}_{|i}$. The random variables $\mathbf{Y}_{-i} \mid Y_i = y$, for $y > u_{Y_i}$ have vector mean and standard deviation given by

$$\boldsymbol{\mu}_{|i}(y) = \mathbf{a}_{|i}(y) + \boldsymbol{\mu}_{|i}\mathbf{b}_{|i}(y) \ ,$$

$$\boldsymbol{\sigma}_{|i}(y) = \boldsymbol{\sigma}_{|i}\mathbf{b}_{|i}(y).$$

Thus $\boldsymbol{\theta}_{|i} = (\mathbf{a}_{|i}, \mathbf{b}_{|i}, \mathbf{c}_{|i}, \mathbf{d}_{|i}, \boldsymbol{\mu}_{|i}, \boldsymbol{\sigma}_{|i})$ are the parameters of a multivariate regression model with non-constant variance and unspecified error distribution. Heffernan and Tawn (2004) take the components of $\mathbf{Z}_{-i}$ to be mutually independent and, for convenience and computational simplicity, they select the Gaussian distribution to obtain point estimates for $\boldsymbol{\theta}_{|i}$. Therefore the objective function

$$Q_{|i}(\boldsymbol{\theta}_{|i}) = -\sum_{j\neq i}\sum_{k=1}^{n_{u_{Y_i}}}\left[\log\{\sigma_{j|i}(y_{i|i,k})\} + \frac{1}{2}\left\{\frac{y_{j|i,k} - \mu_{j|i}(y_{i|i,k})}{\sigma_{j|i}(y_{i|i,k})}\right\}^2\right] \qquad (9)$$

is minimized with respect to all components of $\boldsymbol{\theta}_{|i}$, with $\boldsymbol{\mu}_{|i}$ and $\boldsymbol{\sigma}_{|i}$ being nuisance parameters. We fit the dependence models in two stages: first fixing $c_{j|i} = d_{j|i} = 0$, and only estimating these parameters when if $\hat{a}_{j|i} = 0$ and $\hat{b}_{j|i} < 0$.

There are several sources of uncertainty: from the estimation of the marginal models, of the parametric normalization functions assumed for the dependence structure, and from the nonparametric models for the limit distribution. We carry on semiparametric bootstrap methods to evaluate estimates standard errors. Actually, we assume that the marginals and dependence thresholds are known, and therefore the uncertainty due to threshold selection is not taken into account by the bootstrap methods. The algorithm is based on the following steps:

- The original data are transformed to possess Gumbel margins using (3) and point estimates obtained from the original data.
- A bootstrap nonparametric sample is obtained by sampling with replacement from the transformed data. This step preserves the dependence structure.

- The marginal values of the bootstrap samples are changed ensuring that all marginal distributions are Gumbel and preserving the association between the ranked points in each component.
- The resulting sample is transformed back to the original margins using the marginal models.

The examples worked out by Heffernan and Tawn (2004) showed that the convergence of the conditional distribution of $\mathbf{Y}_{-i} \mid Y_i = y$, as $y \to \infty$, to its limiting form can be very slow. However, as claimed by the authors, we are not interested in the true limit values of $\boldsymbol{\theta}_{|i}$ and $G_{|i}$. What is of practical important is whether the conditional distribution of the normalized variable $\mathbf{Z}_{-i}$ is stable over the range of Y_i values used for estimation and extrapolation. For the marginal models, procedures for checking of the stability of the shape parameter estimates (Coles, 2001) are well known, for example, the mean excess plot. The stability of the estimates of $\boldsymbol{\theta}_{|i}$ may be checked by fitting the conditional model over a range of high thresholds. In addition, formal statistical tests for independence may be applied to the $\mathbf{Z}_{-i}$ to identify whether the variables may be treated as being asymptotically conditionally independent.

After checking the assumptions behind the models, one can focus on the estimation of functionals of the variables defined on regions were data are scarce. Samples of the estimated conditional distribution of $\mathbf{X} \mid X_i > v_{X_i}$, $i = 1, \cdots, d$ are used to estimate the desired functionals. For example, the probability $Pr\{\mathbf{X} \in C_i \mid X_i > v_{X_i}\}$ is estimated using the proportion in the generated sample falling in C_i. To generate the samples we follow the algorithm

1. Simulate y_i^* from the Gumbel distribution conditional on its exceeding $t_i(v_{X_i})$.
2. Sample $\mathbf{z}_{-i}^*$ from $\hat{G}_{|i}$ independently of y_i^*.
3. Obtain $\mathbf{y}_{-i}^* = \hat{\mathbf{a}}_{|i}(y_i^*) + \hat{\mathbf{b}}_{|i}(y_i^*)\mathbf{z}_{-i}^*$
4. Transform $\mathbf{y}^* = (\mathbf{y}_{-i}^*, y_i^*)$ to the original scale using the inverse of transformation (4).
5. Repeat the steps 1-4 to obtain a large simulated sample of the conditional distribution of $\mathbf{X} \mid X_i > v_{X_i}$.

3 Applications of the conditional model in finance

The existing methods for the estimation of extreme financial risk include univariate extreme value models (for example, the generalized extreme value distribution or the generalized Pareto distribution), or some multivariate extreme value model (typically in the bivariate case). The new risk measures estimated in this section are computed under the assumption that at least one component is extreme. We illustrate their truly multivariate nature providing examples in the case $d = 5$.

The data consist in daily closing prices of most important emerging stock markets indexes, from 03/01/1994 through 30/06/2006, collected from the Datastream database. We analyze four Latin American and four Asian emerging markets. The Latin American indexes are the General Index (Argentina), the IBOVESPA (Brazil), the IGPA (Chile), and the IPC (Mexico); and the Asian indexes are the Hang Seng Index (Hong Kong), Bombay Sensitivity Index (India), Seoul Composite (Korea), Taipei Weighted Price Index (Taiwan). To represent the developed countries, we use the S&P500 (U.S.) and the Nikkei Average (Japan). We compute the daily log-returns in U.S. dollars for each index. During the sample period, the Brazilian and the Indian markets outperformed the other ones, as measured by their median log-returns, respectively equal to 0.061% and 0.035%. Brazil is the market presenting higher variability, with a standard deviation of 2.852%, followed by Argentina and Korea. Chile is the least volatile market with a standard deviation of 0.936%, followed by the U.S. market (1.048%).

Among all series, the smallest daily return was observed in Argentina (-28.95%), followed by Mexico and Brazil (-16% and -15%). Brazil presented the highest daily return of 27.05%, followed by Korea and Argentina (22%). All emerging and developed markets indexes may be considered fat tails series, with Argentina being the market presenting the largest excess kurtosis of 12.63%, followed by Hong Kong and Korea (12%).

Financial log-returns are considered stationary series presenting weak autocorrelation in just few lags, and significant autocorrelation in their squares. The may also exhibit long range dependence in the mean and in the volatility. To remove the temporal dependences, we selected the componentwise monthly minima and maxima, resulting in series with length $T = 150$. The monthly data do not present evidence of short and long memory anymore. The hypothesis of a time trend was also tested and rejected for all series. We report only the results for the joint losses. When applying the models to the negative tails we previously multiply the series by (-1) and treat them as maxima.

3.1 Conditional extremal dependence in the Latin American markets

Careful inspection of the data and analysis of marginal GPD fits (Kolmogorov goodness of fit test, Bickel and Doksum, 1981) suggested taking as threshold values u_{X_i}, $i = 1, \cdots, 5$, the empirical 25%, 32%, 32%, 29%, and 31% quantiles of the monthly minima from, respectively, Argentina, Brazil, Chile, Mexico, and U.S. The marginal cdf (3) is then computed for the data. Upper panel of Table 1 shows the threshold values u_{X_i}, the threshold exceedance probabilities $\widehat{F}_{X_i}(u_{X_i})$, the GPD scale and shape parameters estimates $\widehat{\beta}_i$ and $\widehat{\xi}_i$ (and standard errors), and the estimated marginal quantiles $\widehat{x}_i(0.01) = \widehat{F}_{X_i}^{-1}(0.01)$, $i = 1, \cdots, 5$.

The differences among the estimated 0.01-quantiles reflect the different marginal distributions of the left tail of the major indexes in Latin America. The Brazilian and the Mexican markets present close $\widehat{x}_i(0.01)$ values, even though their ξ estimates (and thus, their left tails) are quite different. The Argentinian and the Chilean markets stand out and show the most and less extreme $\widehat{x}_i(0.01)$ values.

We apply transformation (1.4) to obtain the standardized Gumbel marginals. A formal statistical goodness of fit test is applied and accepts the good adherence of the Gumbel distribution to the transformed data. Graphical inspection of histograms with Gumbel density superposed also confirmed the good fits.

Table 1: *Summary of marginal fits to the monthly minima from the Latin American indexes (upper panel) and Asian indexes (lower panel)†.*

	A - Joint Losses - LA				
	Argentina	Brazil	Chile	Mexico	U.S.
u_{X_i}	-5.48	-5.31	-1.84	-4.06	-2.16
$\widehat{F}_{X_i}(u_{X_i})$	0.25	0.32	0.32	0.29	0.31
$\widehat{\beta}_i$	2.07(0.45)	2.71(0.61)	1.26(0.28)	1.73(0.45)	0.62(0.16)
$\widehat{\xi}_i$	0.32(0.19)	0.00(0.11)	-0.32(0.11)	0.32(0.21)	0.31(0.18)
$\widehat{x}_i(0.01)$	-17.14	-14.21	-4.50	-14.56	-5.95
	B - Joint Losses - ASIA				
	Hong Kong	India	Korea	Taiwan	Japan
u_{X_i}	-3.69	-3.59	-4.18	-3.42	-3.08
$\widehat{F}_{X_i}(u_{X_i})$	0.21	0.19	0.33	0.32	0.34
$\widehat{\beta}_i$	1.36(0.43)	1.72(0.50)	1.53(0.30)	1.46(0.31)	1.36(0.32)
$\widehat{\xi}_i$	0.25(0.16)	0.00(0.12)	0.30(0.11)	0.00(0.15)	0.00(0.10)
$\widehat{x}_i(0.01)$	-9.92	-8.63	-13.73	-8.49	-6.37

†Notation in table: u_{X_i} is the threshold used for modeling margin i and the corresponding cumulative probability is $\widehat{F}_{X_i}(u_{X_i})$; the maximum likelihhod estimate and (standard error) of scale is $\widehat{\beta}_i$ and of shape is $\widehat{\xi}_i$; the estimated p-quantiles are $\widehat{x}_i(p)$.

Figure 2 shows the scatter plots of the Gumbel transformed monthly minima for a selection of pairs. The figure illustrates the varying degrees of (unconditional) extremal dependence among the Latin American markets. It seems that asymptotic dependence could be a feasible assumption for plot (b) (Brazil-Mexico), but not for figures (a) (Argentina-U.S.) and (c) (Chile-U.S.). All other pairs may be considered asymptotically independent, with those involving Brazil (except the U.S. case) presenting the strongest level of extremal dependence.

Likewise Heffernan and Tawn (2004), we select the same dependence threshold value for the five indexes, that is, we set $u_{Y_i} = u$ for all i, and choose u such as $\Pr\{Y_i < u\} = 0.30$.

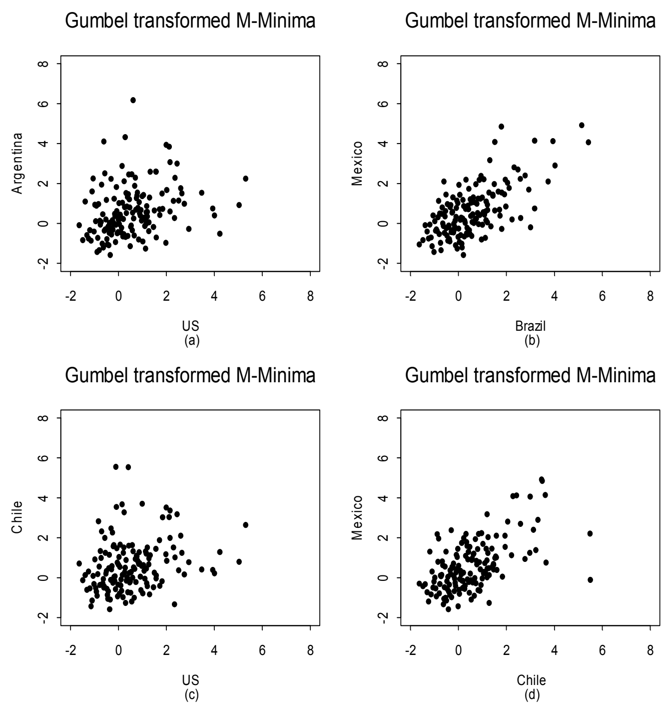

Figure 2: *Pairwise plots of selected Gumbel transformed monthly minima log-returns from Latin American markets.*

We then fit the dependence model (7) consisting of a set of five conditional models, and a total of 20 parameters to estimate.

The estimates $(\widehat{\mathbf{a}}_{\cdot|i}, \widehat{\mathbf{b}}_{\cdot|i})$ are given in Table 2. According to the point estimates, all pairs of minima are asymptotically independent but exhibit positive extremal dependence. This means that the conditional quantiles of X_j tend to $-\infty$ as $y_i \to -\infty$. However, there are two cases of near asymptotic dependence (Brazil | U.S.) and (Mexico | U.S.), as indicated by the $\widehat{a}_{j|5}$, $j = 2, 4$, estimates close to one and $\widehat{b}_{j|5}$, estimates close to zero. Interestingly, these pairs seem to be *not* weak pairwise exchangeable. The pairs presenting strongest extremal dependence are (Chile, Mexico) and those combining Brazil and either Argentina, or Chile, or Mexico.

The convex hulls associated with the point estimates of the dependence parameters are shown in Figure 3, for the pairs involving the U.S. For each fixed (i, j), $i \neq j$, the plots show the pairwise sampling distribution from 100 bootstrap realizations of the sampling distribution of $(\widehat{a}_{i|j}, \widehat{b}_{i|j})$ (solid line), and $(\widehat{a}_{j|i}, \widehat{b}_{j|i})$ (dotted line). For example, in (a) we show the dependence structure between Argentina (1) and U.S. (5) by plotting the sampling distribution of $(\widehat{a}_{1|5}, \widehat{b}_{1|5})$ (solid line), and $(\widehat{a}_{5|1}, \widehat{b}_{5|1})$ (dotted line). We observe the already mentioned pairwise non-exchangeability in plots (b), (c), and (d), and the large variability of estimates, expected since all the $\widehat{b}_{\cdot|5}$ are positive. Note that, given U.S. is extreme, both Brazil and Mexico convex hulls contain the point $(a = 1, b = 0)$. All other pairs (not shown here) indicate weak pairwise exchangeability, but likewise Heffernan and Tawn (2004), we do not attempt to identify a simplified model.

We examine the scatterplots of pairs of components of the estimated limit distributions $\widehat{G}_{|i}$. They indicate that (Brazil, Mexico) for the distribution $\widehat{\mathbf{Z}}_{-5}$, and (Mexico, U.S.) for the distribution $\widehat{\mathbf{Z}}_{-3}$, may be considered tail dependent. All other pairs are asymptotically conditionally independent. A possible interpretation is that Chile drives the dependence between Mexico and U.S. Probably due to the geographical situation, Chile and Mexico experiment some degree of contagion. The interpretation of the asymptotic dependence found for (Brazil, Mexico) given U.S. is extreme is more difficult, since these markets are unconditionally tail dependent, and Mexico was also found being asymptotic dependent on Brazil being extreme. Upper panel of Table 3 summarizes these results.

Table 2: *Parameters estimates of dependence models fitted to the monthly minima of the Latin American (left panel) and Asian (right panel) stock markets indexes.*

	Latin America - Joint Losses				Asia - Joint Losses			
	Bra|Arg	Chi|Arg	Mex|Arg	US|Arg	Ind|HKg	Kor|HKg	Taiw|HKg	Jap|HKg
$\widehat{a}$	0.3480	0.2542	0.0000	0.1521	0.3598	0.7089	0.6565	0.3669
$\widehat{b}$	0.6212	0.2310	0.6438	0.2988	-0.5203	0.4441	-0.1754	0.1909
	Arg|Bra	Chi|Bra	Mex|Bra	US|Bra	HKg|Ind	Kor|Ind	Taiw|Ind	Jap|Ind
$\widehat{a}$	0.7364	0.6194	0.7557	0.2094	0.2235	0.4335	0.3152	0.6887
$\widehat{b}$	0.3120	-0.3388	0.1277	0.3519	-0.2147	0.2509	-0.6332	0.3471
	Arg|Chi	Bra|Chi	Mex|Chi	US|Chi	HKg|Kor	Ind|Kor	Taiw|Kor	Jap|Kor
$\widehat{a}$	0.4004	0.7446	0.7707	0.0000	0.8307	0.3590	0.4525	0.8088
$\widehat{b}$	-0.0684	0.6199	0.6784	0.3234	0.2922	0.3524	-0.1160	0.2466
	Arg|Mex	Bra|Mex	Chi|Mex	US|Mex	HKg|Taiw	Ind|Taiw	Kor|Taiw	Jap|Taiw
$\widehat{a}$	0.6579	1.0000	0.7443	0.0000	0.4380	0.0702	0.0000	0.0000
$\widehat{b}$	-0.1594	0.5001	-0.2579	0.8617	0.4664	-0.1444	0.5835	0.3136
	Arg|US	Bra|US	Chi|US	Mex|US	HKg|Jap	Ind|Jap	Kor|Jap	Taiw|Jap
$\widehat{a}$	0.4850	0.7748	0.4871	0.9182	0.5797	0.5192	0.4058	0.2260
$\widehat{b}$	-0.1324	0.1034	0.0003	0.2827	-0.5196	0.1270	0.2062	-0.0459

Table 3: *Unconditional and conditional interdependences and contagion.*

	Latin America		
Pairs	Unconditionally	Conditionally	Given
(Brazil, Mexico)	Asymp. Depend.	Asymp. Dependent	U.S.
(Mexico, U.S.)	Asymp. Independ.	Asymp. Dependent	Chile
	Asia		
Pairs	Unconditionally	Conditionally	Given
(HKong, Taiwan)	Near Asymp. Independ.	Asymp. Dependent	India or Japan
(India, Japan)	Asymp. Independ.	Asymp. Dependent	Hong Kong

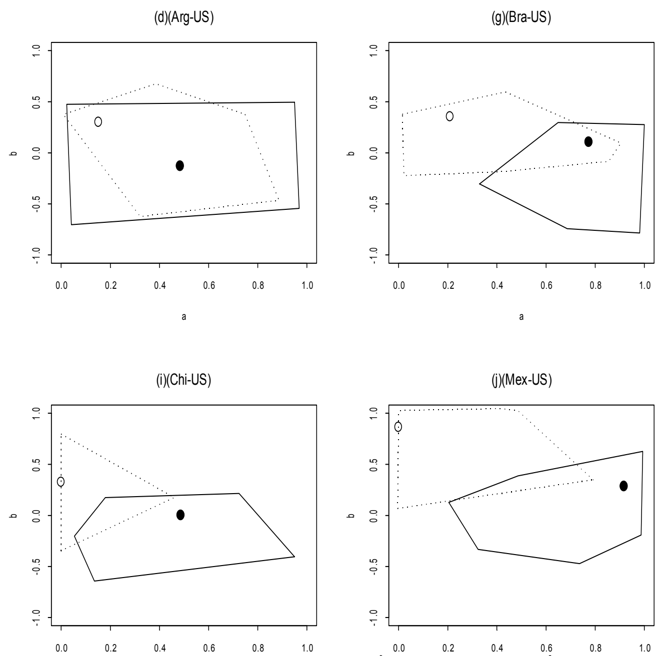

Figure 3: *Bivariate sampling distributions of* $(\widehat{a}_{i|j}, \widehat{b}_{i|j})$ *and* $(\widehat{a}_{j|i}, \widehat{b}_{j|i})$ *for i being the indexes Argentina (1), Brazil (2), Chile (3), Mexico (4), and j being the U.S. (5). The solid line corresponds to* $(\widehat{a}_{i|j}, \widehat{b}_{i|j})$*, and the dotted line corresponds to* $(\widehat{a}_{j|i}, \widehat{b}_{j|i})$.

We now provide our first application of the estimated models, and investigate the effect of the conditional dependence on portfolio selection. We construct two equally weighted portfolios (so called portfolios 1 and 2), composed by (Mexico, U.S., Argentina) and (Mexico, U.S., Chile), and compute their historical accumulated gains. Note that portfolio 1 seems to be a higher risk investment since its Value-at-Risk (VaR) at risks 0.01%, 0.1%, 0.05% values are more extreme than those for portfolio 2, which are, respectively, (-6.86,

-3.87, -2.26), and (-5.17, -2.84, -1.61). Even though Chile presents less extreme losses when compared to Argentina, its strong interdependence during bear markets with U.S. and Mexico, results in smaller accumulated returns for portfolio 2, as we can see in Figure 4.

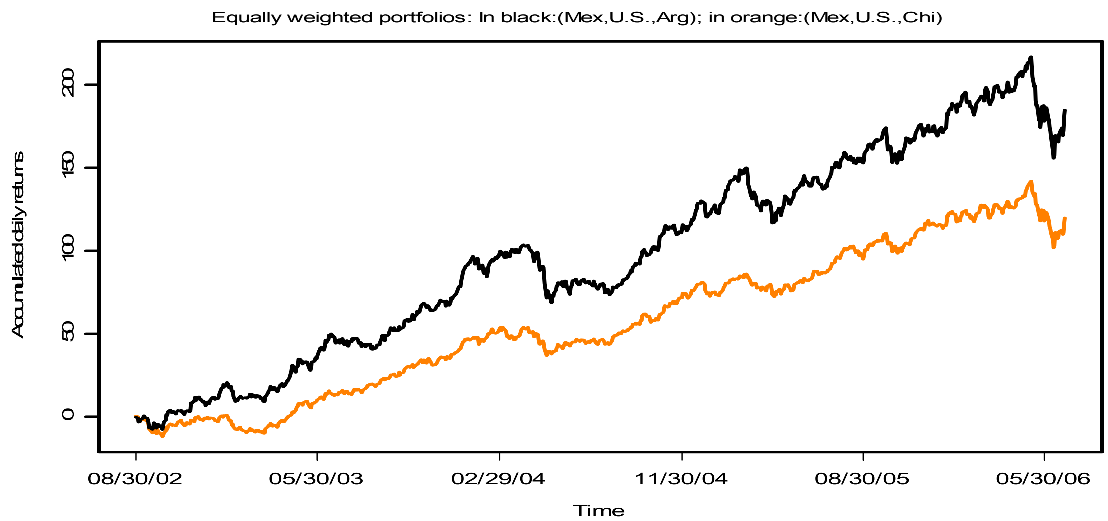

Figure 4: *Accumulated gains from two portfolios. In black (Mexico, US, Argentina), in orange (Mexico, US, Chile).*

To illustrate the influence of conditioning variable on the resulting estimated dependence structure and on the marginal distributions, we show in Figure 5 the pairwise pseudo samples from the estimated conditional models, in the original scale, given that X_2 falls below its $x_2(0.05)$-quantile. On each pairwise plot, the curve represents equal marginal quantiles. The small crosses (+) represent the points that fall in the set $C^5(23)$, that is, the points adding up to 23 in the Gumbel scale. The large filled circles are the 6 points with largest $\sum_{i=1}^{5} y_i$.

In Figure 5 we observe the near asymptotic dependence of Argentina, Chile, and Mexico on Brazil, indicated by the simulated points grouping around the equal quantiles curves. The effect of negative estimate for $b_{3|2}$ is the increasing concentration of this conditional distribution for larger values of the Chilean index. The conditional distribution of U.S. given Brazil is extreme, seems to be a mixture of an near independence component and a tail dependence component. This structure deserves further investigation. We wonder if this could be caused by another variable being extreme, and also if a more robust estimate would identify the predominant component.

A brief summary of the other conditional distributions are as follows: Conditional on Argentina being extreme, Brazil shows weak positive extremal dependence, whereas

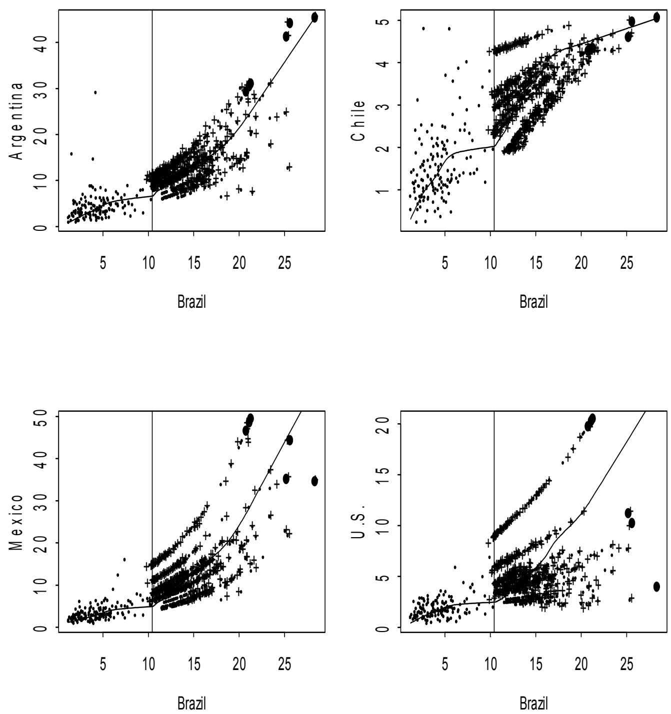

Figure 5: *Data simulated from the estimated conditional models and in the original scale. We show the simulated pairwise conditional distributions, given that the Brazilian index exceeds its empirical quantile $x_i(0.95)$. The vertical lines correspond to the threshold $x_2(0.95)$. Points below and above the thresholds are the original and the simulated data respectively. The "+" represent the points that fall in the set $C^5(23)$. The large filled circles represent the 6 points with the largest values of $\sum_{i=1}^{5} y_i$. The curves represent equal marginal quantiles.*

all other components show near extremal independence. Given Chile is extreme, Argentina shows near extremal independence, and all others show asymptotic independence with positive extremal dependence. Conditional on Mexico being extreme, U.S. shows near independence, Argentina and Brazil weak positive extremal dependence, and Chile asymptotic dependence. Conditional on U.S. being extreme, all markets but Brazil exhibit a mixture distribution.

It would be interesting to have a statistic to measure dependence at extreme levels, such as χ and $\bar{\chi}$, see Ledford and Tawn (1966) and Coles, Heffernan and Tawn (1999). To measure monotone dependence we compute the Kendall's τ coefficient, using the unconditional joint distribution and the simulated conditional distribution. The results are given in Table 4 for some selected pairs. We observe that Brazil does not affect the extremal dependence between Chile and Mexico, as well as Mexico does not affect the pair Brazil and Chile. However, given that the U.S. market is extreme, the monotone interdepence observed for these two pairs increase.

Table 4: *Unconditional and conditional measures of monotone dependence: the Kendall's τ coefficient.*

Brazil & Chile		Chile & Mexico		Hong Kong & India		Korea & Taiwan	
Undondit.	0.4142	Uncondit.	0.3641	Uncondit.	0.2626	Uncondit.	0.2847
Given Mex.	0.2574	Given Bra.	0.2539	Given Kor.	0.0600	Given HKg.	0.4922
Given U.S.	0.5015	Given U.S.	0.5258	Given Jap.	0.6810	Given Jap.	0.5259

Table 5: *Conditional Value-at-Risk computed using the estimated models for Brazil and U.S. given the remaining indexes exceed thei empirical* 0.05*-quantile.*

Conditional Var of Brazil (Losses)					
Risk	|Arg	|Chi	|Mex	|U.S.	Unconditional
1%	-13.66	-16.67	-14.12	-15.78	-14.10
0.1%	-18.11	-23.30	-21.05	-20.09	-15.74
Conditional Var of U.S. (Losses)					
Risk	|Arg	|Bra	|Chi	|Mex	Unconditional
1%	-5.55	-6.35	-5.34	-8.97	-6.32
0.1%	-7.49	-13.00	-6.30	-14.22	-6.86
Conditional Var of India (Losses)					
Risk	|HgKg	|Kor	|Taiw	|Japan	Unconditional
1%	-6.16	-11.98	-6.29	-7.82	-8.09
0.1%	-6.77	-16.63	-7.94	-13.25	-10.63
Conditional Var of Japan (Losses)					
Risk	|HgKg	|Ind	|Kor	|Taiw	Unconditional
1%	-6.41	-7.07	-6.87	-7.03	-6.29
0.1%	-6.92	-8.01	-7.65	-7.26	-6.65

The varying levels of dependence observed among the Latin American markets suggest inspecting the magnitudes of extreme values of a fixed index given that each other in turn is extreme. This is actually the first new risk measure computed, the VaR *conditional* to another variable being extreme, the model based conditional quantile. For example, Table 5 shows the conditional and the unconditional VaR for Brazil and U.S. at the 1% and 0.1% exceedance probabilities. For both markets, the VaR values computed using the estimated models are more extreme when compared to the unconditional empirical VaR, reflecting the positive extremal dependence existing among the components. We observe that the worst scenarios for Brazil occur when either Chile or U.S. are extreme, and for the U.S. occur when either Brazil or Mexico are extreme.

We introduce here our second new risk measure, the Conditional Expected Shortfall of risk α, defined as the $\mathrm{E}[X_j|X_i < x_i(\alpha)]$, α a small probability, $i, j = 1, \cdots, 5$. We compute the thresholds $x_i(\alpha)$ as the empirical α-quantiles of the original data and set $\alpha = 0.05$ and $\alpha = 0.01$. This is a different risk measure since we compute the expected loss for some variable, given that another one is extreme, or exceeds a particular level. It may be considered a generalization of the well known Expected Shortfall, recovered when $i = j$. In Table 6 we provide estimates for this quantity, conditional on U.S. falls below its thresholds -3.26 and -4.73. For $\alpha = 0.05$ we compare the model based expectations to the empirical ones. For $\alpha = 0.01$ the empirical values are not reliable and are not used. Since all variables are positively associated, the conditional expectations increase as we consider higher quantiles.

Table 6: *Empirical and model-based estimates of Conditional Expected Shortfall for the Latin American losses, given that U.S. falls below its* 0.05 *and* 0.01*-quantiles, in the upper panel; and for the Asian losses, given that Japan falls below its* 0.05 *and* 0.01*-quantiles, in the lower panel.*

Latin America - Joint Losses				
X_j	$\mathrm{E}[X_j]$	$\mathrm{E}[X_j \mid X_5 < x_5(0.05)]$		$\mathrm{E}[X_j \mid X_5 < x_5(0.01)]$
	Empirical	Empirical	Model based	Model based
Argentina	-4.51(0.27)	-6.94(0.63)	-9.14(0.16)	-10.72(0.11)
Brazil	-4.84(0.22)	-8.93(0.54)	-11.85(0.29)	-12.94(0.19)
Chile	-1.66(0.08)	-2.49(0.25)	-3.10(0.09)	-3.16(0.10)
Mexico	-3.56(0.21)	-7.40(0.73)	-12.40(0.77)	-16.50(0.80)
U.S.	-1.86(0.08)	-5.07(0.23)	-6.91(0.18)	-10.28(0.23)
Asia - Joint Losses				
X_j	$\mathrm{E}[X_j]$	$\mathrm{E}[X_j \mid X_5 < x_5(0.05)]$		$\mathrm{E}[X_j \mid X_5 < x_5(0.01)]$
	Empirical	Empirical	Model based	Model based
Hong Kong	-2.83(0.15)	-5.39(0.76)	-8.49(0.73)	-8.90(0.88)
India	-2.82(0.13)	-5.29(0.96)	-6.61(0.38)	-7.61(0.39)
Korea	-3.72(0.21)	-8.87(1.23)	-14.62(0.81)	-14.99(1.43)
Taiwan	-3.10(0.14)	-3.89(0.51)	-5.42(0.61)	-5.97(0.69)
Japan	-2.82(0.10)	-6.03(0.13)	-6.52(0.19)	-7.04(0.49)

Another appealing and useful novelty provided by the model is the computation of the conditional VaR of a portfolio, given that one of their components (or some other variable in the model) is extreme. This is equivalent to estimate return levels of linear combinations of the Gumbel transformed variables. The Gumbel scale emphasizes the effect of dependence on extreme combinations, and facilitates the comparisons. We consider the five-dimensional sets $C^5(v)$ defined as $C^5(v) = \{(y_1, y_2, y_3, y_4, y_5) \in \Re^5 : \sum_{i=1}^{5} y_i < v\}$, for some extreme negative value v. For a fixed probability α, we report the estimated return level v_α — the Conditional VaR — the value implicitly defined by $\Pr\{\mathbf{Y} \in C^5(v_\alpha)\} = \alpha$, which depends upon the conditional dependence model and the marginal models.

To illustrate, we show in Figure 6 the Conditional model based VaR of an equally weighted portfolio given that variable i is extreme, $i \in \{2, 5\}$, that is, given that Brazil is extreme, and given that the U.S. is extreme. For the exceedance probability 1%, the VaR values are respectively -3.994 and -3.858; and for the 5% risk the values are -2.849 and -2.614, reflecting the important influence of Brazil in the Latin America.

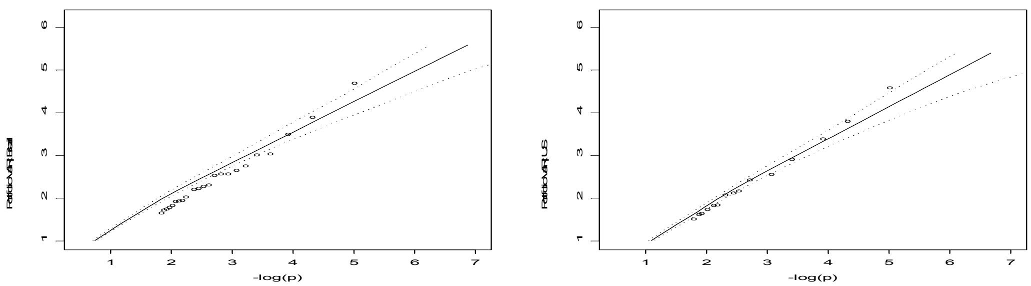

Figure 6: *Conditional Value-at-Risk of a equally weighted portfolio e conditional on Brazil and U.S., respectively, being extreme, for joint losses in Latin America. The return level is* $\sum_{i=1}^{5} y_i$*, represented by the solid line. Dotted lines correspond to the 0.95 confidence interval. The points "o" represent the empirical return levels. The horizontal axis has the logarithm of the exceedance probabilities.*

3.2 Conditional extremal dependence in the Asian markets

We now use the emerging Asian markets to illustrate the applications of the conditional model. All the inference steps carried on in the previous section are repeated for the 5 major Asian indexes. The marginal and dependence models point estimates are shown in tables 1 and 2. The statistical variability of marginal estimates are similar to those observed for the Latin American markets. The Asian markets considered seem to have shorter left tail when compared to the Latin American ones, as indicated by the marginal 0.99-quantiles. The plots of the Gumbel transformed variables show different levels of unconditional extremal dependence, being all pairs asymptotic independent. The sampling distributions of the pairs $(\widehat{a}_{i|j}, \widehat{b}_{i|j})$ and $(\widehat{a}_{j|i}, \widehat{b}_{j|i})$, for all i, j, indicate weak pairwise exchangeability with the convex hulls showing large intersections.

The scatterplots of pairs of components of $\mathbf{Z}_{-i}$, for all i, reveal that India-Japan are conditionally asymptotically dependent when Hong Kong is extreme, and that Hong Kong-Taiwan may be considered asymptotically conditionally dependent given either India or Japan is extreme. Interesting to note that *all* pairs are asymptotically unconditionally independent. A possible interpretation is that the interdependence observed at high quantiles between India and Japan may be a contagion of a crisis disseminated by Hong Kong. In the case of the pair Hong Kong and Taiwan the contagion is driven by either India or Japan. These results are summarized in Table 3.

By examining the estimates $(\widehat{a}_{i|j}, \widehat{b}_{i|j})$, for all $i \neq j$, we observe that there is a strong dependence between the markets Hong Kong and Korea, with evidence that they might be near asymptotically dependent. All (i, j) pairs of estimates indicate positive association, with (Taiwan, Japan) close to independence.

Figure 7 illustrates the different degrees of dependence and show pseudo-samples on the measured scale from the conditional distribution of the remaining variables given that India is extreme. The conditional distributions of Hong Kong and Korea, given India is extreme, may be considered as mixtures of an independent component and extremal positive dependence. According to Heffernan and Tawn (2004) this may induce bias in the estimates. Japan is asymptotically dependent, but Taiwan shows near asymptotic independence.

Figure 8 shows pseudo-samples on the measured scale from the conditional distributions of India given each one of the remaining variables is extreme. It seems that all of them are mixtures, have a component related to a near asymptotic independence, and another related to positive extremal dependence. Large values are observed when either Korea or Japan is extreme. On the other hand, Japan is also affected by India. Given India is extreme, we observe tail dependence with Japan. Given Hong Kong is extreme we observe near independence for Japan.

To help understanding the effect of this conditional dependence we construct again two equally weighted portfolios composed by 1: (Hong Kong, Taiwan, India), and 2: (Hong Kong, Taiwan, Korea). Note that portfolio 2 includes Korea, a market which does not imply tail dependence on the pair (Hong Kong, Taiwan). The VaR-0.01%, 0.1%, 0.05% values for portfolios 1 and 2 are, respectively, (-5.74, -3.07, -1.77), and (-6.79, -3.64, -2.13). We compute the historical accumulated gains of the two portfolios, and observe that even though portfolio 2 presents more extreme VaR values, it shows higher accumulated gains higher than those from portfolio 1, probably due the extreme losses presented by the combination in portfolio 1. This deserves a more comprehensive investigation, as the behavior of a portfolio also depends on the marginal distributions.

We show in Table 5 the Conditional VaR of India and of Japan. The effect of the different levels of positive conditional dependence is seen by comparing these figures to the unconditional VaR. Korea is the market affecting India the most, and India is the one affecting Japan the most.

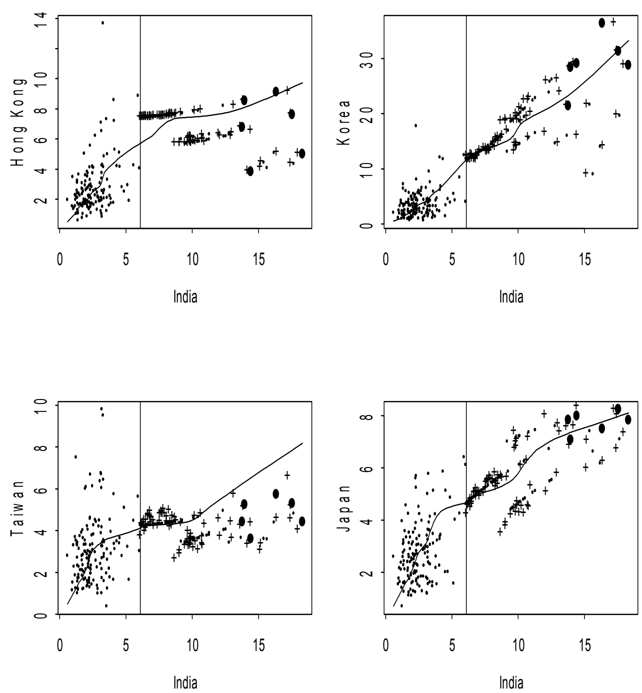

Figure 7: *The simulated pairwise conditional distributions, given that India exceeds its empirical quantile* $x_i(0.95)$*. The vertical lines correspond to the threshold* $x_2(0.95)$*. Points below and above the thresholds are the original and the simulated data respectively. The "+" represent the points that fall in the set* $C^5(23)$*. The large filled circles represent the 6 points with the largest values of* $\sum_{i=1}^{5} y_i$*. The curves represent equal marginal quantiles.*

Finally, using the pseudo-samples we compute the Conditional Expected Shortfall, shown in Table 6. They are the mean loss of all markets given that Japan has fallen bellow its empirical 0.05- and 0.01-quantile. A portfolio manager may be interested in assessing these functionals under the different conditional distributions to help selecting portfolio components.

4 Discussion

In this paper we have exploited the conditional model proposed by Heffernnan and Tawn (2004), providing another application of the new extreme value model. The investigation carried out here may help to better understand the strengths and applicability of the

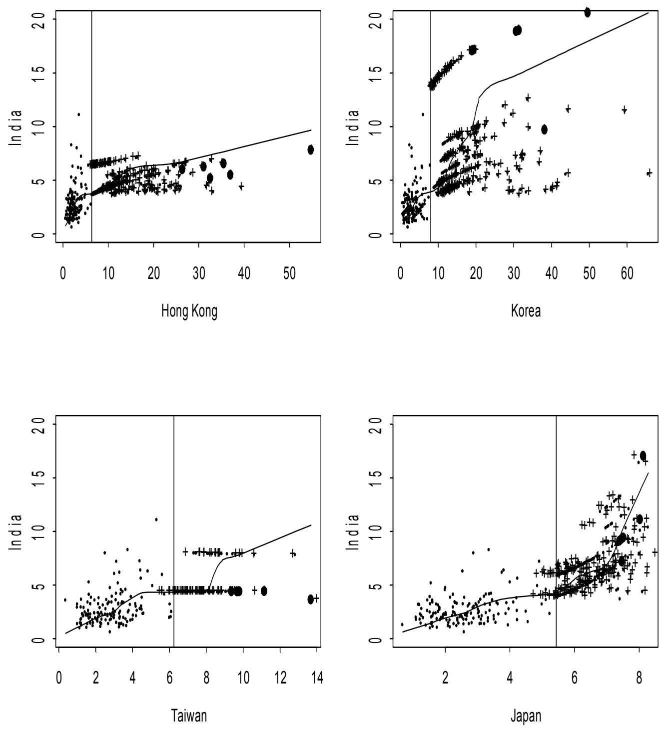

Figure 8: *Tthe simulated pairwise conditional distributions for India given that each market has exceeded its empirical quantile $x_i(0.95)$. The vertical lines correspond to the threshold $x_i(0.95)$. Points below and above the thresholds are the original and the simulated data respectively. The "+" represent the points that fall in the set $C^5(23)$. The large filled circles represent the 6 points with the largest values of $\sum_{i=1}^{5} y_i$. The curves represent equal marginal quantiles.*

multivariate conditional model, while providing new insights on the complex behavior of emerging stock markets. One of the contributions of the modeling strategy is the computation of the new conditional risk measures.

We found that many of the conditional distributions possess two different components, which may be related to other economic and political aspects. One of the components act as if the variables involved were independent, whereas the other one implies positive extremal dependence. The phenomenon of contagion may also contribute on this kind of behavior. As discussed in the original paper, the mixture distributions my result in bias in the estimators. For example, we observed that for Brazil and Argentina, just a few points seemed to have been generated by the near independence component. We wonder if robust estimates, using their main property which is to identify the distribution followed by the majority of the data, in this case the near asymptotic dependent component, would provide better estimates, and would reflect the worst cases scenarios for interdependence in Latin America. We have replaced the Gaussian model by the t-student distribution, but this did not lead to better fits.

For this particular application, we did not address the issue of robustness of results to changes on data collection. In a future work we may investigate whether or not results substantially change if we take bi-weekly or bi-monthly componentwise minima.

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